IMPLICATIONS OF DIFFERENT DESIGN ASSUMPTIONS IN DIRECT DISPLACEMENT BASED DESIGN OF RC FRAMES

BEATRIZ MASSENA
PhD student
ICIST, IST
Lisbon – Portugal

RITA BENTO
Associate Professor
ICIST, IST
Lisbon - Portugal

HERVÉ DEGÉE
Research Associate
Département ArGEnCo
Université de Liège
Belgique

SUMMARY

Through the last years, as the importance of displacements, rather than strength, has come to be better appreciated, various contributions were made towards the development of displacement-based seismic design. Nevertheless only in the 1990's formal proposals were made to implement the emerging ideas into a design procedure. These methodologies are based on the determination of an optimal structural strength to achieve a given performance limit state, defined as a given damage level under a specified level of seismic intensity. The aim of this paper is to assess a particular design procedure so-called as Direct Displacement-based Design procedure, i.e. an evaluation of the actual performance level of structures designed according to DDBD. All the steps of DDBD methodology will be clearly investigated and discussed along with the implications of the assumptions made in each step for designing reinforced concrete structures. The main design options considered are vertical distribution of column moments, storey shear distribution, horizontal distribution of bending moments in beams, as well as the consequences of the design for gravity loads in these elements. The methodology to be followed will be first used to design a set of structures according to DDBD with different assumptions, and then to assess the seismic behaviour of the designed structures by means of non-linear static analyses and non-linear dynamic time-history analyses. Additional consideration is also made on the impact of equivalent viscous damping formulas. Conclusions are dealing with an identification of the different assumptions having the most significant impact on the final design, as well as with practical recommendations for the designer.

1. INTRODUCTION

The Direct Displacement-Based Design is one of the displacement-based seismic design methodologies that emerged in the 1990's as a result of the growing interest for methods based on displacements, in particular for what regards RC structures. The DDBD procedure was developed on the base of Priestley's works and aims at designing structures in order to achieve displacements corresponding to a specified limit state under earthquake action. In this method, it is necessary to define the required strength at designed plastic hinge locations in order to obtain the targeted structural performance level under design earthquake. Capacity design rules are then applied to guarantee that plastic hinges do not occur in other regions than the desirable locations, avoiding the development of non-ductile modes of inelastic deformation.

In DDBD, the design process for a multi-degree of freedom structure (MDOF) starts with the determination of the characteristic of an equivalent single-degree-of-freedom (SDOF) structure (representation of performance at peak displacement response). This equivalent SDOF is based on the "substitute structure" analysis procedure [1] and is characterized by the secant stiffness K_e at maximum displacement Δ_d and by a level of equivalent viscous damping ξ representing the combined effect of viscous and hysteretic dissipation. The main governing parameters of equivalent SDOF structure are given on Fig.1, where K_i stands for the initial elastic stiffness, r is the ratio of post-yield to initial stiffness, Δ_y is the yield displacement, μ is the ductility, F_{max} and F_y are the maximum force and yield force, respectively. After knowing the SDOF characteristics, it is possible to determine the design base shear V_{base}. The design base shear obtained from the SDOF structure is then distributed as equivalent inertia lateral forces in the original structure (MDOF structure). The design moment at potential plastic hinges are then determined as well as the design moments and shears for all the others critical structural sections.

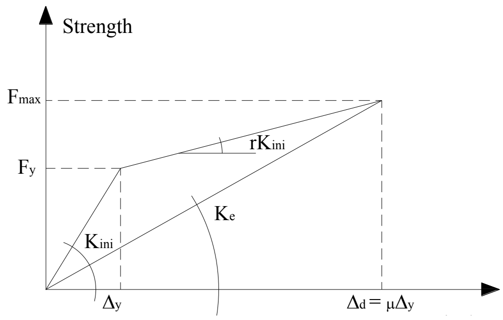


Figure 1: Constitutive law of the SDOF system

2. DDBD METHOD FOR REINFORCED CONCRETE FRAMES

2.1. Overall summary of the procedure

In the following is presented a brief description of DDBD procedure:

Step 1: Definition of the target displacement shape and amplitude of the MDOF structure on the base of performance level considerations (material strain or drift limits) and then derive from there the design displacement Δ_d of the substitute SDOF structure (see Fig.2, in which δ_i are the design displacements of the MDOF structure).
Step 2: Estimation of the level of equivalent viscous damping ξ. The equivalent viscous damping can be obtained by one of the equations proposed in the technical literature [2, 3 and 4]. To obtain the equivalent viscous damping, the displacement ductility μ must be known. The displacement ductility is the ratio between the design displacement and the yield displacement Δ_y. The yield displacement is estimated according to the considered properties of the structural elements, for example through the use of approximated equations proposed by Priestley [2] based on the yield curvature.
Step 3: Determination of the effective period T_e of the SDOF structure by using the design displacement defined in step 1 and the design displacement response spectrum corresponding to the damping level estimated in step 2.
Step 4: Derivation of the effective stiffness K_e of the substitute SDOF structure from its effective mass m_e and effective period. The design base shear V_{Base} is the product of the effective stiffness by the design displacement.
Step 5: Distribution of the design base shear vertically and horizontally to the structural elements of the lateral load resisting system (frames and/or walls).
Step 6: Assessment of moment capacities at potential hinge locations. To this purpose, two different methods of analysis can be used according to Ref. [2], one is based on relative stiffness members while the other is a simplified method based on equilibrium considerations (statically admissible distribution of internal forces). Herein only the latter is described.

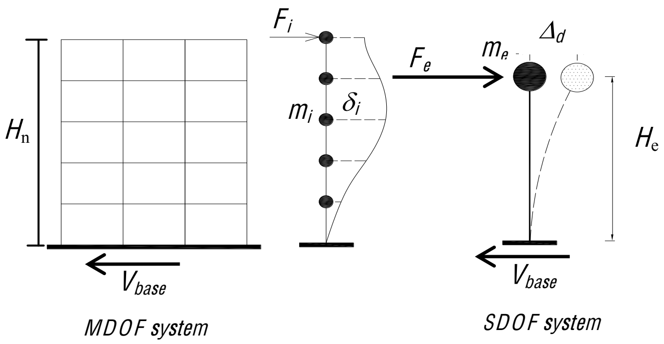


Figure 2: Simplified model of a multi-storey building

2.2. Analysis based on Equilibrium considerations

2.2.1. Beam Moments

Fig.3 presents a typical distribution of seismic lateral forces resulting from the DDBD procedure and the corresponding internal forces induced in a frame building.

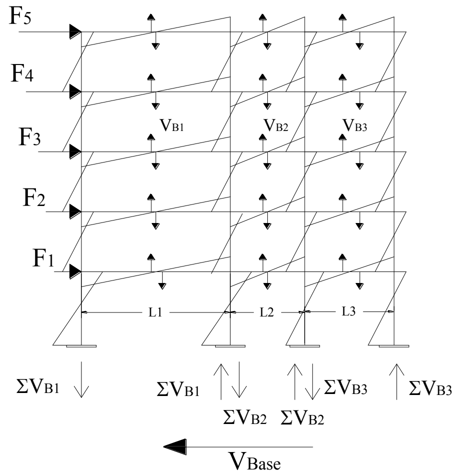


Figure 3: Seismic Moments from DDBD adapted [2].

The lateral seismic forces F_i produce in each of the columns axial forces (compression or tension) and column-base moments (M_c). The seismic axial forces induced in each of the columns by the seismic beams shears (V_{Bi}) are $\sum V_{Bi}$. Considering the equilibrium at base level, the total overturning moment is given by:

$$OTM = \sum_{i=1}^{n} F_i H_i \tag{1}$$

where H_i is the height of floor i.

Knowing that equilibrium should be maintained between internal and external forces, the total overturning moment at the base of the structure is thus:

$$OTM = \sum_{j=1}^{m} M_{cj} + \sum_{i=1}^{n} V_{Bi} \times L_{Bi} \tag{2}$$

where M_{cj} are the column-base moments (m columns) and L_{Bi} is the length of span i.

Therefore, considering only the parcel of OTM regarding the seismic axial forces (OTM^*), in the particular case of Fig.3:

$$OTM^* = \sum_{i=1}^{n} V_{Bi} \times L_{Bi} = \sum_{i=1}^{n} V_{B1} \times L_1 + \sum V_{B2} \times L_2 + \sum V_{B3} \times L_3 \tag{3}$$

where, V_{B1}, V_{B2} and V_{B3} are the beam shears from bay 1, 2 and 3, respectively.

According to the Fig.3, at a floor i, the beam seismic shears for each span will be given by:

$$V_{Bi} = \frac{2M_{Bi}}{L_{Bi}} \tag{4}$$

Replacing Eq. (4) in Eq. (3) the overturning moment OTM^* is:

$$OTM^* = 2\left(M_{B1} + M_{B2} + M_{B3}\right) \tag{5}$$

Considering a relationship between beam moments as $M_{B2} = \alpha M_{B1}$ and $M_{B3} = \beta M_{B1}$ and replacing in turn in Eq. (5), the beam moments corresponding to the first span are given by the following equation:

$$M_{B1} = \frac{OTM^*}{2(1+\alpha+\beta)} \tag{6}$$

Knowing that the seismic axial forces induced in each of the columns is the sum of beam shears ($\sum V_{Bi}$), for the first span (i=1), according to Eq. (4) and replacing α and β in Eq. (6):

$$\sum_{i=1}^{n} V_{B1,i} = \frac{2M_{B1}}{L_{B1}} = \frac{OTM^*}{L_{B1}}\left(\frac{M_{B1}}{M_{B1}+M_{B2}+M_{B3}}\right) \tag{7}$$

The seismic axial forces for each alignment due to OTM^* are then given by:

$$\sum_{i=1}^{n} V_{Bi} = \frac{M_{Bi}}{\sum_{i=1}^{n} M_{Bi}} \frac{OTM^*}{L_{Bi}} \tag{8}$$

Combining Eq. (1) with Eq. (2) and replacing the parcel of seismic axial forces due to OTM^* given by Eq. (8), the total sum of seismic axial forces is defined in the following:

$$\sum_{i=1}^{n} V_{Bi} = \frac{M_{Bi}}{\sum_{i=1}^{n} M_{Bi}}\left(\sum_{i=1}^{n} F_i H_i - \sum_{j=1}^{m} M_{cj}\right) / L_{Bi} \tag{9}$$

Every distribution of the total required beam shear that assures Eq. (9) will result in a statically admissible equilibrium solution and can be chosen on the base of engineering judgment. In Ref. [5] it is suggested however that the distribution of the total beam shear force could be done in proportion to the storey shears in the level below the beam under consideration as presented in Fig.4.

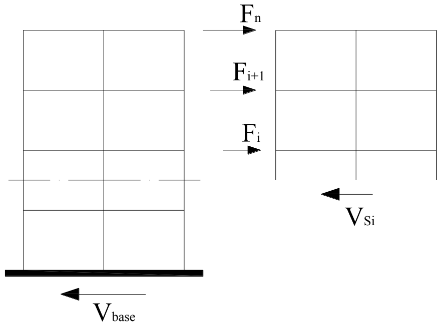


Figure 4: Storey shear forces

The distribution of the total beam shear force is thus defined according to Eq. (10).

$$V_{Bi} = \sum_{i=1}^{n} V_{Bi} \cdot \frac{V_{S,i}}{\sum_{i=1}^{n} V_{S,i}} \tag{10}$$

where the storey shear forces at level i, $V_{S,i}$ are given by:

$$V_{S,i} = \sum_{k=i}^{n} F_k \tag{11}$$

When the shear forces in each beam have been calculated, the beam design moments at the column centrelines are defined by:

$$M_{Bi,l} + M_{Bi,r} = V_{Bi}.L_{Bi} \tag{12}$$

where, $M_{Bi,l}$ and $M_{Bi,r}$ are the beam moments at the column centrelines at the left and right end of the beam, respectively.

2.2.2. Column Moments

Knowing the beam moments, the columns moments can be obtained directly by equilibrium considerations: the total storey shear force [Eq. (11)] is shared between the columns. From the shear forces at the base of each column V_C, it is then possible to obtain the moment at the base and top of the columns, $M_{C1,b}$ and $M_{C1,t}$ respectively. Knowing that structural analysis based on equilibrium considerations is actually an approximation of the real distribution, the designer gets some freedom in choosing the moment capacities at the column-base of first floor, provided the equilibrium is maintained between internal and external forces. In technical literature some suggestions are made to estimate the moment capacities of the column-base hinges, Ref. [2]. To obtain the moments in the whole columns, the procedure must then be continued with consideration of equilibrium at the node of level 2 and successively until the top level is reached.

3. ASSESSMENT OF DDBD PROCEDURE

In the aforementioned summary of the DDBD procedure it is clear that some design assumptions suggested in the technical literature are still under the responsibility of the design engineer (e.g. position of the contra-flexure point in the columns or sharing of the horizontal storey shear between the columns). Moreover, technical literature proposes different formulas to evaluate the equivalent viscous damping, among which the choice should be done by the designer (although some are more recommended than others). Therefore, the present paper outlines a preliminary study aiming at investigating the impact of different design assumptions on the final design of RC frames and on their seismic performance. To this purpose a simple case-study is designed according to the different assumptions and five situations are considered:

- Case 1: DDBD procedure according to all design assumptions suggested by Priestley [2];
- Case 2: DDBD procedure considering that horizontal shear is shared between the columns according to their bending stiffness;
- Case 3: DDBD procedure assuming that contra-flexure point in the columns is always at mid-height, including for columns of the bottom storey;
- Case 4: DDBD procedure the same as case 1 but using Blandon-Priestley formula to estimate the equivalent viscous damping [3];
- Case 5: DDBD procedure the same as case 1 but using Dwairi-Kowalsky formula to estimate the equivalent viscous damping [4].

Results are expressed in terms of design bending moment at beam and column ends, as well as in terms of required longitudinal reinforcements. Finally, the performance of the five designed structures is assessed by means of non-linear static analyses and non-linear dynamic time-history analyses.

3.1. Case-study

3.1.1 Description and design assumptions

The DDBD procedure is applied to the interior frame of the four-storey reinforced concrete structure plotted as section A-A of Fig.5, with a global geometry (height and spans as well as beams and columns cross-section dimensions) defined in the context of the "Cooperative Research on the Seismic Response of the Reinforced Concrete Structures" [6]. The structure is irregular in terms of spans and the lateral resistance is provided by one-way frame action. The slab thickness is equal to 0.15 m and its contribution to the structural response was taking in account by considering an effective beam width according to Eurocode 8 [7]. To model more adequate the soil-foundation-structure interaction rotational springs were defined. The reinforced concrete frames are made with concrete C25/30 (f_{cd} = 16.7 MPa). The reinforcement steel is a classical Tempcore steel B500 (f_y =500 MPa). In addition to the self-weight of the beams and the slab, a distributed dead load of 2 kN/m^2 due to floor finishing and partitions is considered, as well as an imposed live load with nominal value of 2 kN/m^2. In situations where they have to be combined with seismic actions, dead loads are considered as their nominal value and live loads as 40% of their nominal value.

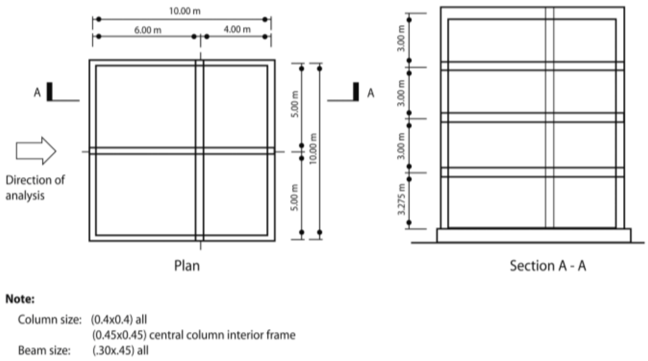


Figure 5: General Layout [adapted from Ref. [6]]

The frame building is considered being located in Continental Portugal (Algarve) as an ordinary building class of importance II. The seismic action is defined by Eurocode 8 [7] and Portuguese National Annex [8] with the elastic acceleration response spectrum S_a for subsoil class D. The value of the peak ground acceleration a_g used in the definition of the response spectrum is 0.35g. The elastic displacement spectrum S_{De} used for DDBD, shown in Fig.6, is the one defined in Eurocode 8 by:

$$S_{De}(T) = S_a(T)\left[\frac{T}{2\pi}\right]^2 \tag{13}$$

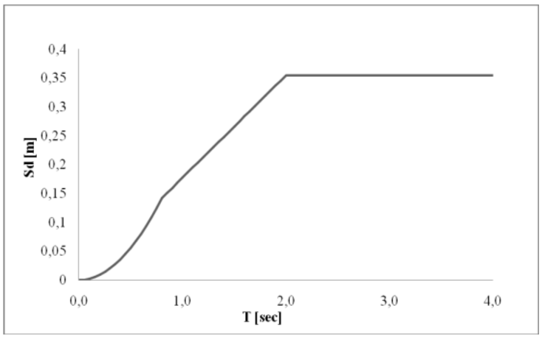


Figure 6: Design Displacement Spectrum

According to Priestley [2], for frame buildings the design displacement will usually be governed by a specified drift limit in the lower storeys of the building. Therefore, in this case of study, for a damage control limit state (Level 2), the drift limit was considered as 2.5 %.

3.2. Results from DDBD procedure

3.2.1. Global parameters

In Fig.7 is shown the design displacements profile according to the selected target drift limit, where the top target displacement Δ_{target} (roof displacement) is equal to 0.307m (MDOF structure - see section A-A of Fig.5). Knowing the design displacements profile it is straightforward to derive corresponding characteristics of the equivalent SDOF structure, for which the design displacement Δ_d is equal to 0.230 m. The effective period at peak displacement response is found from the design displacement spectra, the latter is defined for the equivalent viscous damping ξ corresponding to the assumption followed for each design case, i.e. entering the design displacement Δ_d and determining the effective period T_e. It is finally possible to obtain the effective stiffness K_e and base shear V_{Base}. The values obtained are given in Table 1, where it can be seen that the results are rather independent from the chosen damping equation.

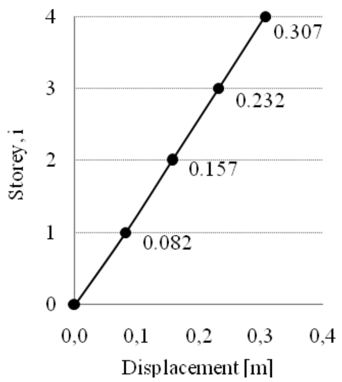


Figure 7: Design storey displacements

Table 1 – Results of DDBD in terms of displacement, ductility, effective mass and effective period and base shear

Design situation	Δ_{target} (m)	Δ_d (m)	Δ_y (m)	μ	m_e (tonne)	ξ (%)	T_e (s)	V_{Base} (kN)
Case 1	0.307	0.230	0.141	1.64	157.34	11.99	1.69	498.70
Case 2								
Case 3								
Case 4						12.11	1.70	495.33
Case 5						13.05	1.75	469.64

3.2.1 Internal forces

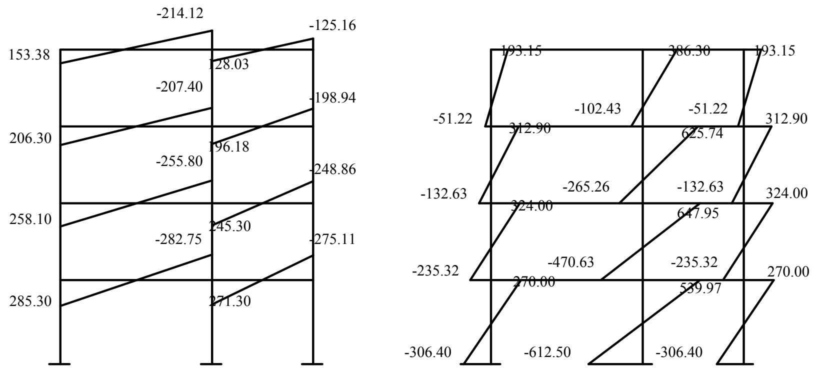
In Fig.8 is presented the design values of bending moments obtained for beams and columns, respectively, for case 1 to 5. Table 2 shows the longitudinal reinforcement for columns for cases 1 to 5. The beam moments are obtained from direct analysis (using statically admissible distribution for DDBD), while column moments are obtained from capacity design considerations. The capacity design is performed according to Priestley [2] recommendations. The required column flexural strength according to DDBD capacity design rules is given by:

$$\phi_f M_N \geq \phi^0 \omega_f M_E \tag{14}$$

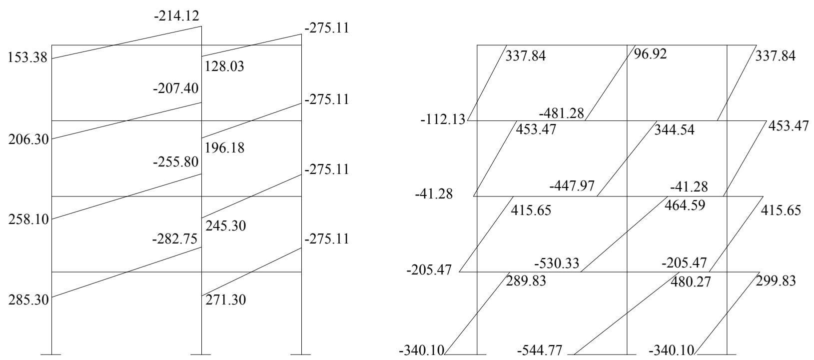
where,
ϕ^0 is the overstrenght factor relating maximum feasible flexural strength to design strength, herein considered as 1.25
ω_f is the dynamic amplification factor, height and ductility dependent
M_E is the column moments resulting from lateral seismic forces (see Fig.3)
M_N is the design column moments, presented in Fig.8
ϕ_f is the strength reduction factor, herein considered as 0.9

The design beam moments are obtained as the most challenging situation between gravity loads and seismic loads considered separately according to Ref. [2] suggestions.

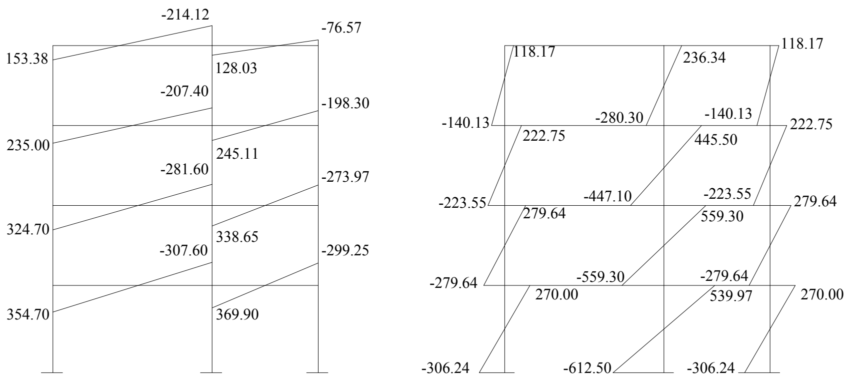
a) Case 1



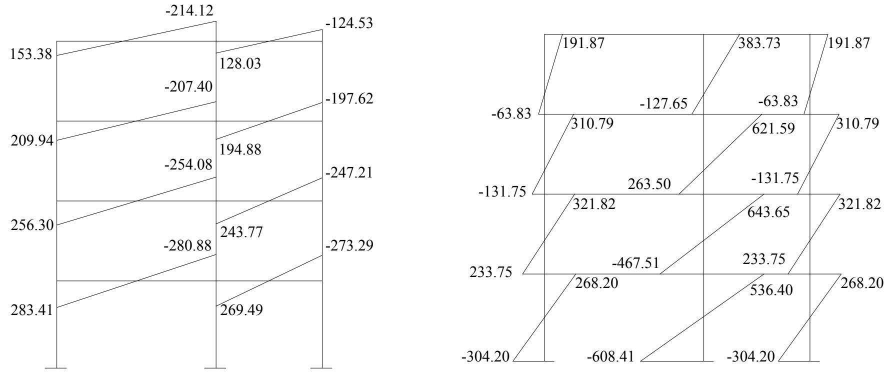
b) Case 2



c) Case 3



d) Case 4



d) Case 5

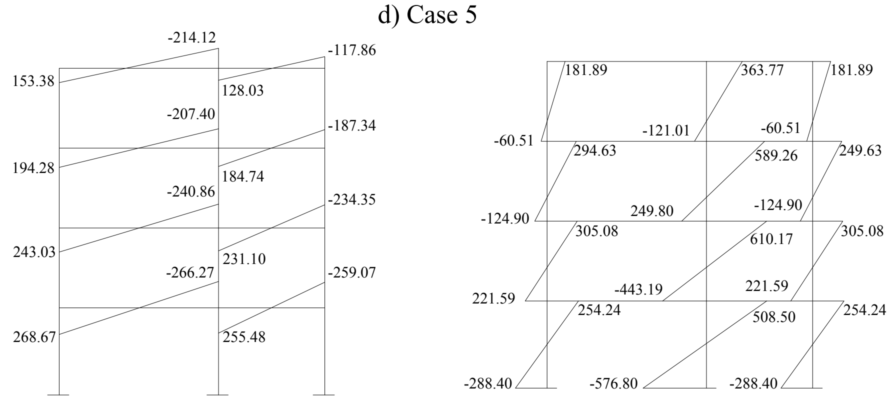


Figure 8: Internal forces, design beam and column moments - Results of DDBD [units in kNm]

Table 2 – Longitudinal reinforcement for columns - Cases 1 to 5

Nº of rebar's per face	Case 1 and Case 4			Case 2			Case 3			Case 5		
	Col 1	Col 2	Col 3	Col 1	Col 2	Col 3	Col 1	Col 2	Col 3	Col 1	Col 2	Col 3
4th floor	$3\phi25$	$3\phi32$	$3\phi25$	$5\phi25$	$3\phi32$	$5\phi25$	$3\phi25$	$3\phi32$	$3\phi25$	$4\phi25$	$3\phi32$	$3\phi25$
3rd floor	$4\phi25$	$5\phi32$	$4\phi25$	$6\phi25$	$3\phi32$	$6\phi25$	$3\phi25$	$3\phi32$	$3\phi25$	$4\phi25$	$4\phi32$	$4\phi25$
2nd floor	$4\phi25$	$5\phi32$	$4\phi25$	$4\phi25$	$4\phi32$	$5\phi25$	$4\phi25$	$4\phi32$	$3\phi25$	$4\phi25$	$4\phi32$	$4\phi25$
1st floor	$4\phi25$	$4\phi32$	$3\phi25$	$4\phi25$	$3\phi32$	$3\phi25$	$4\phi25$	$4\phi32$	$3\phi25$	$4\phi25$	$3\phi32$	$3\phi25$

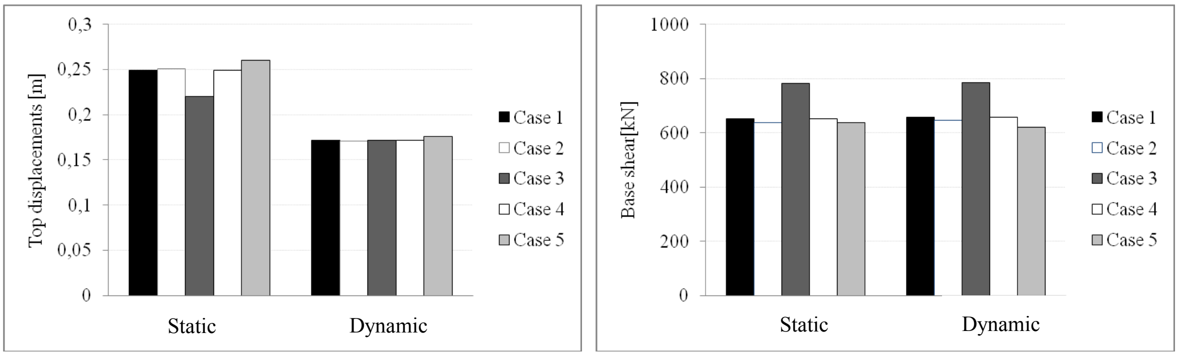
3.2.2 Performance assessment

According to the design moments of Fig.8, reinforcement schemes have been selected (see Table 2) and the criteria for ductile behaviour of concrete sections defined in Eurocode 8 fulfilled (Ductility Class Medium DCM, see Ref. [7]). The five resulting structures have then been submitted to non-linear static (Pushover) analyses and to non-linear dynamic time-history analyses with the aim of assessing their performance under seismic conditions. To this purpose, a set of seven artificial ground motion time histories compatible with the EC8 spectrum (defined in section 3.1) has been generated with the software GOSCA [9]. Numerical simulations have then been carried out with Seismostruct [10] using fibre beam elements.

In the following, results from both non-linear analyses (static and dynamic) are presented in terms of top displacements (roof displacements), base shear – Table 3 and Fig.9 a) and b), respectively. These results have to be related to the target top displacement considered from the very beginning of the DDBD procedure (i.e. 0.307 m, see Fig.7).

Table 3 – Top displacement and corresponding base shear – assessment of the designed structures

	Top displacement (m)			Base shear (kN)		
	Static	Dynamic		Static	Dynamic	
		max	aver.		max	aver.
Case 1	0.249	0.195	0.172	651.43	732.72	658.60
Case 2	0.251	0.197	0.171	637.62	713.60	645.90
Case 3	0.220	0.190	0.172	780.95	805.50	783.60
Case 4	0.249	0.195	0.172	651.43	732.72	658.60
Case 5	0.260	0.227	0.176	637.60	688.75	619.90



a) Top displacement b) Base shear forces

Figure 9: Results from non-linear static and dynamic analyses

From Fig.9 a) and b) it can be stated that top displacements for the different cases are quite insensitive to design assumptions. The main difference occurs for base shear in case 3 that exceeds the values of the other cases of about 18%, for both non-linear static and dynamic analyses. It can be stated that cases 1 and 4 led to the same results due to equal reinforcement schemes; therefore, in the following, only the results for case 1 are presented.

Fig.10 depicts the interstory drifts obtained with non-linear static and non-linear dynamic analyses. The maximum values occur in the first storey for the case studies 1, 2, 4 and 5 as far as non-linear dynamic analyses concerned. The configuration of the interstory drifts for the lower stories obtained for the case studies 1, 2, 4 and 5 is different from the expected configuration for a frame building, i.e. the interstory drift is no decreasing along the height of the building. This effect is mainly due to the characteristics of the beams, as they are too flexible and thus do not give in a proper rotation restriction for columns at the storey levels, but also due to the soil-structure interaction model considered. Therefore different types of soil-structure interaction models were initially adopted: built-in columns and flexible supports by means of different rotation springs (see Fig.11). The results obtained were compared; for flexible rotation springs the interstory drifts will decrease in height as expected for frame structures. The results herein presented correspond to models where base flexible supports are adopted and the values of stiffness of the rotational springs K_{Θ} were defined taking into account that the seismic action was defined for subsoil class D.

In case 3 the configuration of interstory drift is decreasing with the height. The main difference between case 3 and the other cases of study regards with the distribution of column moments. In all models, except case 3, the column bending moments were obtained by equilibrium considerations, node by node, considering the values of the beam moments obtained previously. In case 3 the column moments distribution along height were obtained firstly admitting equal column moments at top and bottom as $M_{ci}=0.5V_{Ci}H_i$, except at base were the moment capacities of the column-base hinges are obtained according the suggestion in Ref. [2], and then by equilibrium considerations the beam moments were obtained. In case 3 and according to the chosen distribution of internal moments the bending moments in the beams increase in the lower stories (see Fig 8).

Nevertheless, the interstory drift values obtained with non-linear static analyses are always conservative.

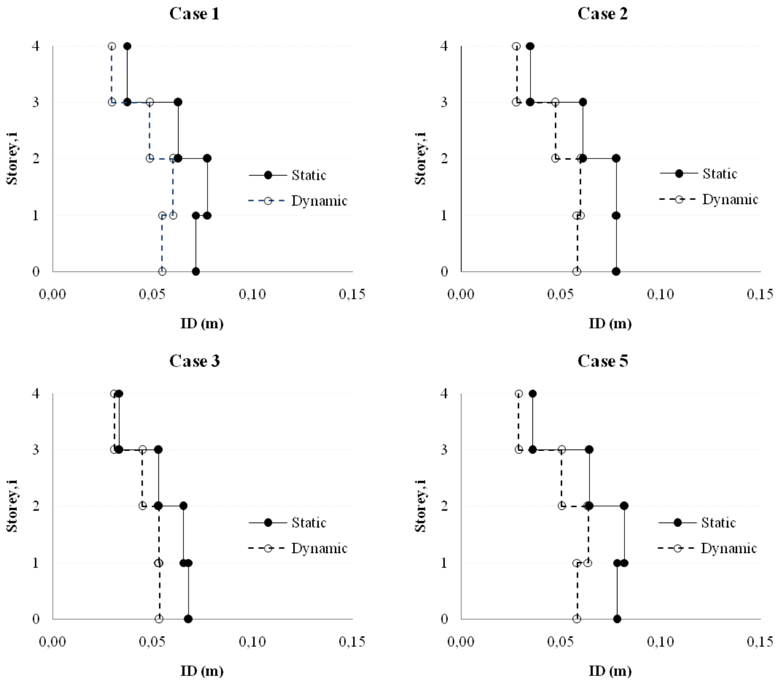
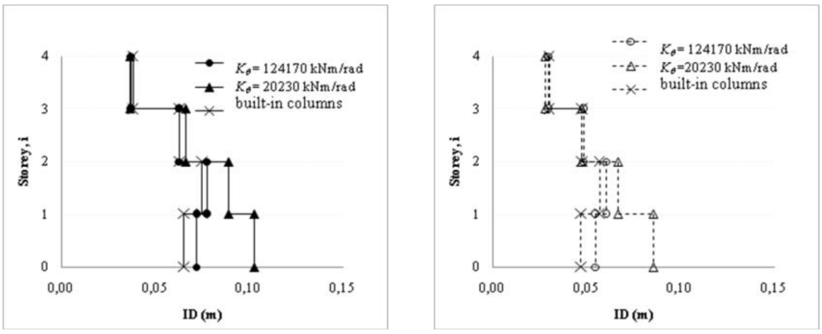


Figure 10: Interstory drifts - Case 1 to 5



a) Non-linear static analyses b) Non-linear dynamic analyses

Figure 11: Interstory drifts for different support conditions - Case 1

Fig.12 shows the distribution of plastic hinges in the structure from the non-linear dynamic time-history analyses and found identical for the five study cases and the 7 accelerograms. The distribution of plastic hinges shows that there are formations of plastic hinges were intended and not in other locations than the expected ones (in the base columns and at beam ends); i.e. the weak beam-strong column criterion was fulfilled.

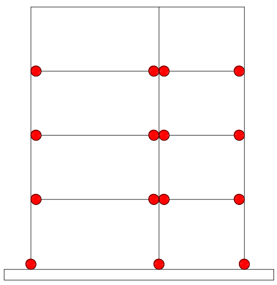


Figure 12: Plastic hinges distribution for non-linear dynamic time-history analyses - Case 1 to 5 for the 7 accelerograms

4. CONCLUSIONS

In this paper, some results of the assessment of a simple case–study are presented to illustrate the consequences of design assumption that must be fixed by the designer in the context of DDBD procedure.
In this particular example, it can be seen that the only assumption that has a significant influence on the final design is dealing with the position of the contra-flexure point in columns. However, this has only a marginal impact of the final structural behavior and it could then be suggested to use in practice the distribution that minimizes the quantity of reinforcements. Another interesting conclusion is that final design does not significantly depend on the formula used to derive the equivalent damping. The suggestion should then be to use the simplest one (i.e. proposed by Priestley in Ref [2]).
Finally, it is seen that, whatever the design assumptions, seismic assessment of the final design leads to the development of an inelastic mechanism corresponding to the expected one (i.e. plastic hinges formation at beam ends and base of the columns). It can however been remarked that the target performance level is not reached and that the structure is actually stiffer than expected.

5. AKCNOWLEDGEMENT

The authors would like to acknowledge the financial support of the Portuguese Foundation for Science and Technology (Ministry of Science and Technology of the Republic of Portugal) through the research project PTDC/ECM/100299/2008 and through the PhD scholarship SFRH/BD/36648/2007 granted to B. Massena. H. Degée thanks for the support received from F.R.S.-FNRS (Belgian Fund for Research).

6. REFERENCES

[1] Shibata, A., and Sozen, M. (1976) Substitute structure method for seismic design in RC. Journal of Structural Division ASCE 102(ST1), 1-18.7

[2] Priestley, M. N. J., Calvi, G.M. and Kowalsky, M.J. (2007) Displacement-Based Seismic Design of Structures. *IUSS Press*, Pavia, Italy.

[3] Blandon C.A., Priestley M.J.N. (2005) Equivalent viscous damping equations for direct displacement design. *Journal of earthquake Engineering, No 9 (special issue 2)*, 257-278.

[4] Dwairi, H.M., and Kowalsky, M.J. (2004) Investigation of Jacobsen's Equivalent Viscous Damping Approach as applied to Displacement-Based Seismic Design. *13th World Conference on Earthquake Engineering – 13WCEE*, Aug. 1-6, Vancouver, Canada. Paper No 228.

[5] Pettinga, J.D. and Priestley, M.J.N. (2005) Dynamic behavior of reinforced concrete frames designed with direct displacement-based design. Journal of Earthquake Engineering. Vol 9 (2), 309-330.

[6] CRSRRCS 1992, Negro, P., Varzeletti, G., Magonette, G. E., Pinto, A., V. (1995) Tests on the Four-Storey Full Scale Reinforced Concrete Frame with Masonry Infills: Preliminary Report, *Joint Research Centre-Ispra, Italy.*

[7] EN 1998-1:2004. Eurocode 8: Design of Structures for Earthquake Resistance - Part 1: General Rules. Seismic Actions and Rules for Buildings. CEN. Brussels. Belgium.

[8] NP EN 1998-1:2007. Portuguese National Anexx.

[9] Denoel. V. (2001) Generation of Spectrum Compatible Accelerograms. Research Report. Université de Liège.

[10] SeismoSoft (2010). SeismoStruct - A Computer Program for Static and Dynamic Nonlinear Analysis of Framed Structures. Available from URL: www.seismosoft.com.